Brief Announcement: Timing Games and Shared Memory

Zvi Lotker[1], Boaz Patt-Shamir[2], and Mark R. Tuttle[3]

[1] CWI, lotker@cwi.nl.
[2] Tel Aviv University, boaz@eng.tau.ac.il.
[3] HP Labs, mark.tuttle@hp.com.

Abstract. We model a simple problem in advertising as a strategic timing game, and consider continuous and discrete versions of this game. For the continuous game, we completely characterize the Nash equilibrium for two players. For the discrete game, we give an efficient algorithm to compute the Nash equilibrium for n players.

We consider a model with a single shared register, a stream of readers arriving at a constant rate in the interval $[0, 1]$, and a set of writers that each write to the register one time. We interpret the value of the register as an advertisement, the readers as customers, and the writers as advertisers. The register represents any public space such as an ad on a web page or an entry in a system directory. The register's value might be a link to a web site or a pointer to some service, resource, or product. Since the advertiser's goal is to maximize the number of customers seeing its ad, and since the customers arrive at a constant rate, this is equivalent to maximizing the length of time the ad is in the register before it is overwritten by another ad. Since each advertiser can write its ad to the register only once, when should it write?

We model this problem as a non-cooperative, complete-information strategic game, a game we call the *publicity game*. The publicity game is a symmetric game with n players, each player i chooses a number $x_i \in [0, 1]$, and the payoff to player i is the distance from x_i to the next point. Formally, given the choices $x_1, \ldots, x_n$ of the players, we define $\text{bigger}(i) = \{x_j \mid x_j \geq x_i \text{ and } j \neq i\}$ and $\text{next}(i) = \min \text{bigger}(i)$. The payoff for player i is defined by

$$u_i(x_1, x_2, \ldots, x_n) = \begin{cases} \text{next}(i) - x_i & \text{if bigger}(i) \neq \emptyset \\ 1 - x_i & \text{otherwise} \end{cases}$$

Note that if two players happen to choose the same value, the payoff to both of them is 0. We consider other payoffs to colliding players in the full paper.

Our publicity game is a new variant of "timing games" like "War of Attrition" [2, 3, 5], but our payoffs do not change with time as in that game, and the work of Baye *et al.* [1] is also related to some of our results for the two-player game. Our game superficially resembles the Hotelling location games [4], but these games either are zero-sum or they involve pricing, and are fundamentally different.

The publicity game has only mixed-strategy equilibria:

Theorem 1. *There is no equilibrium of pure strategies for the publicity game.*

It is not hard to bound the game value of symmetric equilibria:

Theorem 2. *If v is a symmetric equilibrium value for the publicity game with n players, then $1/(n+1) < v < 1/n$.*

In fact, as we now show, we can compute this game value quite accurately, and prove it is unique.

We cannot compute the game value for n players exactly, but we can find a closed form for the special case of two players.

Theorem 3. *There is a Nash equilibrium for the two-player publicity game defined by*

$$f_1(x) = \begin{cases} \frac{1}{1-x} \ , & \text{if } \ 0 \leq x \leq 1 - \frac{1}{e} \\ 0 \ , & \text{otherwise} \end{cases}$$

This equilibrium in unique up to a set of points in $[0,1]$ of measure zero, and the expected payoff for each player is $\frac{1}{e}$.

We can approximate the game value for n players if we restrict attention to a discrete version of the publicity game that restricts players to choosing one of the $k+1$ points $\left\{0, \frac{1}{k}, \frac{2}{k}, \ldots, 1\right\}$. In the full paper, we give an algorithm $\textsc{Equilibrium}(n,k,\epsilon)$ that approximates the symmetric equilibrium value v to within ϵ by doing a binary search for v and inductively computing the probabilities the symmetric equilibrium assigns to the discrete points in the unit interval.

Theorem 4. *There is a unique symmetric equilibrium to the discrete publicity game, and $\textsc{Equilibrium}(n,k,\epsilon)$ computes the equilibrium value to within ϵ.*

This algorithmic approach to finding the equilibrium for the n-player game is interesting, because the differential equations that characterize the equilibria to general n-player games are often quite hard to solve.

This paper is a first step toward understanding the effect of delayed actions on the outcome of timely games. These games arise naturally in many situations such as recommendation systems and other economic systems. To this end, we have defined and analyzed a simple game we called the publicity game. To the best of our knowledge, this is the first time this game is explicitly addressed.

References

1. M. R. Baye, D. Kovenock, and C. de Vries. A general linear model of contests. Mimeo, 1998. Unpublished manscript available from `http://www.nash-equilibrium.com/baye/Contests.pdf`.
2. D. Fudenberg and J. Tirole. *Game Theory.* MIT Press, 1991.
3. K. Hendricks, A. Weiss, and C. Wilson. The war of attrition in continuous time with complete information. *International Economic Review*, 29(4):663–680, Nov. 1988.
4. H. Hotelling. Stability in competition. *Economic Journal*, 39:41–57, 1929.
5. J. Maynard Smith. The theory of games and the evolution in animal conflicts. *Journal of Theoretical Biology*, 47:209–221, 1974.